General relativity is easier than you think

Let's suppose you know almost nothing about general relativity. As a curious reader of popular science you do know some things, but your knowledge boils down to a few well-trod cliches.

You do have a better grasp of special relativity. You understand what time dilation means and how it relates to the universality of the speed of light. You did high school physics, or else have learned the essentials elsewhere.

You might sometimes make a humorous quip about everything travelling forward in time at one second per second. But you know it's not just a joke, because in a relativistic world where time flows at different rates for different observers, I might travel through time at one-and-a-bit seconds for every second of yours.

You do know that general relativity has something to do with spacetime being curved. You understand that near a massive object, time and space exchange roles to some extent, so that an object that would travel through time but not through space travels slightly less through time and slightly more through space, and that's why gravity happens.

As a consequence of this you know that an object in a gravitational field experiences a slower flow of time. Perhaps you forget sometimes whether it flows slower or faster, but you're able to think it through and figure it out.

Perhaps you've watched the video at http://laughingsquid.com/gravitational-attraction-simply-explained-using-a-sheet-of-graph-paper-attached-to-a-crank/ which explains the time and space thing better than I can do in words.

So, you do know a *little* about general relativity, but you consider your knowledge to be insignificant, a few hazy waves of the hand, not real understanding.

Congratulations. You're ready for a quiz.

Calculate the number of years that a hermit would have to spend on the apex of Mount Everest to experience one more second than an observer standing on the beach.

No, you can't have a formula. I want you to *guess* the formula. You already have all the information you need.

Here's what you do. Call the point on the apex a, and the point at the base b. There must be some ratio between the amount of time that passes at a and the amount of time that passes at b, which you can write as $\frac{\Delta t_a}{\Delta t_b}$. Time passes more slowly at b (i.e. b experiences fewer seconds), so this ratio is greater than one.

In non-relativistic high-school physics, you learned there's a thing called gravitational potential energy, equal to mass times height times gravitational acceleration. In relativity there isn't really any such thing as potential energy (blame Emmy Noether for that), but the theory has to give *approximately* the same answer as Newtonian physics, so potential energy must still be a useful concept in the approximate case. Since I haven't asked a question about black holes or anything, you are *allowed* to make a few approximations.

A falling object gives up some of its motion through time in exchange for extra motion through space, so you can think of potential energy as the energy an object possesses on account of experiencing the flow of time. (We're assuming non-relativistic speeds for everything: that's one of our approximations.) Given what you already know about general relativity, you should expect that if an observer at b calculates the potential energy of an object at a, it will be some function of $\frac{\Delta t_a}{\Delta t_b}$, the relative flow of time.

Assume the difference in gravitational acceleration between a and b is negligible. From your high school equation for potential energy, gravitational acceleration (written as g) must *also* be a function of $\frac{\Delta t_a}{\Delta t_b}$ and inversely proportional to the difference in height between a and b (which I'll write as Δh).

So, g is some function of $\frac{\Delta t_a}{\Delta t_b}$ and inversely proportional to Δh. *What function*?

The stronger the gravitational field, the more time passes at a for every second at b, so g increases as $\frac{\Delta t_a}{\Delta t_b}$ increases. But they're obviously not proportional, because time flows at the same rate everywhere when there's no gravitational field at all — i.e. $\frac{\Delta t_a}{\Delta t_b} = 1$ when $g = 0$. Feeling William of Ockham's gaze down the centuries, we choose the simplest equation that is consistent with what we know, and assume g is proportional to $1 - \frac{\Delta t_b}{\Delta t_a}$.

Calling our proportionality constant k, we can write this as:

$$g = \frac{k}{\Delta h}(1 - \frac{\Delta t_b}{\Delta t_a})$$

Now we just need to work out what k is, and if you learned *anything* from high school physics, it's that you start by figuring out what its units must be. On the left, the units of g are metres per second squared (ms^{-2}). On the right, the units are m^{-1} multiplied by the units of k. So k must have units of m^2s^{-2}, which means it is the square of a velocity.

Since we're talking about relativity, where as everyone knows the speed of light is ubiquitous, it doesn't take a great leap of imagination to suppose that $k = c^2$.

Therefore: $g = \frac{c^2}{\Delta h}(1 - \frac{\Delta t_b}{\Delta t_a})$

This equation, remember, *is a guess*. We didn't derive it from first principles. We just figured out the simplest formula that *could possibly be right*, given a few elementary facts.

Back to that question about Mount Everest.

- Acceleration due to gravity: $g = 9.8\ ms^{-2}$
- Speed of light squared: $c^2 = (3\times10^8)^2 = 9\times10^{16}\ m^2s^{-2}$
- Height of Mount Everest: $\Delta h = 8848\ m$

We need to find the value of Δt_a such that $\Delta t_a = \Delta t_b + 1$.

The equation gives us $9.8 = \frac{9\times10^{16}}{8848}(1 - \frac{t-1}{t})$. If you type solve for t 9.8 = (9E16 / 8848) * (1 - (t-1)/t) into Wolfram Alpha, you'll get an answer of about 1.04×10^{12} seconds.

Divide by the number of seconds in a year, and you get about thirty three thousand years.

Believe it or not, that answer is correct. It's *approximate*, as we've noted several times, but it is also *correct*, in full agreement with the answer you'd get from the actual equations used by Einstein.

And you thought you didn't know anything about general relativity.